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1 Derivation of Lawson Criterion for D-T

We are interested in how long the plasma contains energy. We use the quantity τ_E, called the 'confinement time'. This is a measure of the rate at which the system loses energy to the environment. It can be defined as the energy content of the plasma divided by the power loss.

$$\tau_E = \frac{W}{P_{loss}} \tag{1}$$

Where W is the energy content of the plasma, and P_{loss} is the power loss. The thermal energy of a plasma can be defined as:

$$W = \int \frac{3}{2} k(n_{e^-} T_{e^-} + (n_D + n_T) T_{ions}) dV \tag{2}$$

Where k is Boltzmann's constant, n_{e^-} is the electron density, T_{e^-} is the electron temperature, n_D and n_T are the ion densities of deuterium and tritium respectively and T_{ions} is their temperature. The integral is over the volume. If we assume that all temperatures are the same, and that the densities of tritium and deuterium are equal, we get:

$$\frac{W}{V} = 3 n_e k_B T \tag{3}$$

Where in this equation, n_e is the electron density and V is the volume. The number of fusions per volume per time is given by:

$$f = n_D n_T \langle \sigma v \rangle = \frac{1}{4} n_e^2 \langle \sigma v \rangle \tag{4}$$

Where f is the number of fusions per volume per time, n_D and n_T are the ion densities of deuterium and tritium respectively, σ is the fusion cross-section, v is the relative velocity. The angle brackets mean an average over the Maxwellian velocity distribution. We are also assuming that the densities of the ions are equal, and that their total density is equal to the total electron density n_e. This implies that $n_D = n_T = \frac{1}{2} n_e$. For ignition state, we require that the same amount of energy be produced and kept within the plasma as that which leaves the plasma. The rate of heating per volume is defined as the product of f and E_{ch}, the energy of the charged fusion products. We are assuming here (quite accurately) that the neutron emmissions contribute nothing to plasma heating. So we require:

$$f E_{ch} \geq P_{loss} \tag{5}$$

Subbing in our definitions of f and P_{loss} we get:

$$\frac{1}{4} n_e^2 \langle \sigma v \rangle E_{ch} \geq \frac{3 n_e k_B T}{\tau_E} \tag{6}$$

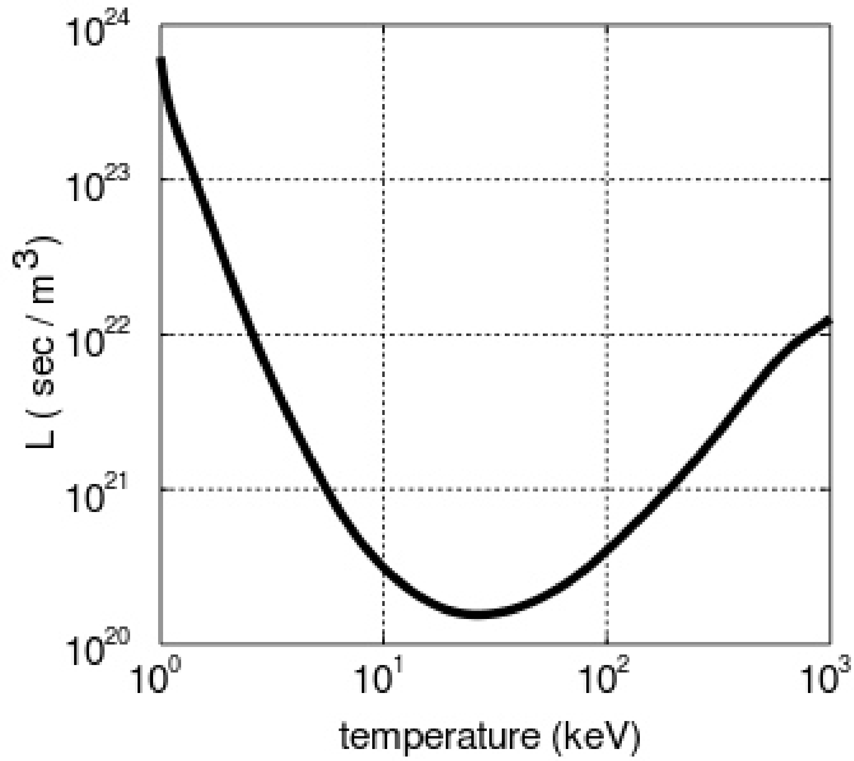
Rearranging leads to the standard definition of the Lawson Criterion.

$$n_e \tau_E \geq \frac{12}{E_{ch}} \frac{k_B T}{\langle \sigma v \rangle} \tag{7}$$

The right hand side of this equation is called the L function. Where:

$$L = \frac{12}{E_{ch}} \frac{k_B T}{\langle \sigma v \rangle} \tag{8}$$

The minimum value of L for D-T is reached near the temperature of 25 keV. This image, now modified slightly, was originally from wikimedia commons.



For D-T the absolute minimum for the product $n_e \tau_E$ is:

$$n_e \tau_E \geq 1.5 \ \times 10^{20} s/m^3 \tag{9}$$

Primary resources for this derivation were:
http://www-fusion-magnetique.cea.fr/gb/fusion/physique/demo_ntt.htm
http://en.wikipedia.org/wiki/Lawson_criterion